SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)#	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)#	–	–	–	**2(2)**
4.	Determinants	1(1)	1(2)*	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	1(1)*	1(2)*	1(3)	–	**3(6)**
8.	Application of Integrals	1(1)	1(2)	1(3)	–	**3(6)**
9.	Differential Equations	1(1)	1(2)	1(3)*	–	**3(6)**
10.	Vector Algebra	3(3)#	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	2(2)#	1(2)	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2) + 1(4)	1(2)	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.
#Out of the two or more questions, one/two question(s) is/are choice based.

MATHEMATICS

Time allowed : 3 hours *Maximum marks : 80*

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Find the magnitude of the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

OR

Find the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.

2. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

3. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

OR

Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class [0].

4. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

5. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, then find the value of 'a' and 'b'.

OR

Write the element a_{23} of a 3×3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$.

6. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, then find the value of x.

7. Find the value of $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$.

OR

Evaluate : $\int_2^4 \frac{x}{x^2+1} dx$

8. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B, state whether f is one-one or not.

9. If a line makes an angle θ_1, θ_2 and θ_3 with the x, y and z-axes respectively, then find the value of $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$.

OR

If the equation of a line AB is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, then find the numbers to which direction ratios of a line parallel to AB, are proportional.

10. Find the order and degree of $y = px + \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$.

11. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

12. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

13. Check whether the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive.

14. If A is a square matrix such that $A^2 = A$, then show that $(I - A)^3 + A = I$.

15. Find the value of $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A \mid B) = \frac{2}{5}$.

16. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 400$ and $|\vec{a}| = 4$, then find $|\vec{b}|$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Neelam and Ved appeared for first round of an interview for two vacancies. The probability of Neelam's selection is 1/6 and that of Ved's selection is 1/4.

Based on the above information, answer the following questions :

(i) The probability that both of them are selected, is

(a) $\frac{1}{12}$ (b) $\frac{1}{24}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

(ii) The probability that none of them is selected, is

(a) $\frac{2}{7}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{3}$

(iii) The probability that only one of them is selected, is

(a) $\frac{5}{8}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{1}{3}$

(iv) The probability that atleast one of them is selected, is

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$

(v) Suppose Neelam is selected by the manager and told her about two posts P and Q for which selection is independent. If the probability of selection for post P is $\frac{1}{6}$ and for post Q is $\frac{1}{7}$, then the probability that Neelam is selected for at least one post, is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$

18. Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm.

Based on the above information, answer the following questions :

(i) If x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm, then x must lie in

(a) [0, 18] (b) (0,9)

(c) (0, 3) (d) None of these

(ii) Volume of the open box formed by folding up the cutting corner can be expressed as

(a) $V = x(18 - 2x)(18 - 2x)$ (b) $V = \frac{x}{2}(18 + x)(18 - x)$

(c) $V = \frac{x}{3}(18 - 2x)(18 + 2x)$ (d) $V = x(18 - 2x)(18 - x)$

(iii) The values of x for which $\frac{dV}{dx} = 0$, are

(a) 3, 2 (b) 0, 3 (c) 0, 9 (d) 3, 9

(iv) Sonam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(a) 13 cm (b) 8 cm (c) 3 cm (d) 2 cm

(v) The maximum value of the volume is

(a) 144 cm^3 (b) 232 cm^3 (c) 256 cm^3 (d) 432 cm^3

PART - B

Section - III

19. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

20. Find $\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$.

OR

Evaluate : $\int \frac{\sin 2x}{\sin\left(x-\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)}dx$

21. Prove that the area enclosed between the x-axis and the curve $y = x^2 - 1$ is $\frac{4}{3}$ sq. units

22. Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$.

23. If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$, find $\vec{a}\times\vec{b}$ and $|\vec{a}\times\vec{b}|$.

OR

Prove by vector method that the area of ΔABC is $\frac{a^2 \sin B \sin C}{2\sin A}$.

24. If A and B are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then find $P(\text{not } A \text{ and not } B)$.

25. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

26. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then find the value of 'a'.

27. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

28. If A is a non-singular 3×3 matrix such that $|5 \cdot \text{adj}A| = 5$, then find $|A|$.

OR

If there are two values of a which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then find the sum of these values.

Section - IV

29. Find the value of k, for which

$$f(x)=\begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$

is continuous at $x = 0$.

30. Evaluate : $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)dx$

31. Differentiate the function $\log\left(\frac{a+b\sin x}{a-b\sin x}\right)$ with respect to x.

OR

Find $\frac{dy}{dx}$, when $y=\sqrt{a+\sqrt{a+\sqrt{a+x^2}}}$, where a is a constant.

32. Find area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

33. If the equation of tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$, then find the values of a and b.

34. Solve the differential equation :

$(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$, subject to the initial condition $y(0) = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y + y\cos y}$, given that $y=\frac{\pi}{2}$, when $x = 1$.

35. Show that the function $f: (-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

Section - V

36. Find the inverse of the matrix $A=\begin{bmatrix} 8 & 4 & 2 \\ 2 & 9 & 4 \\ 1 & 2 & 8 \end{bmatrix}$.

OR

If $A=\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}, B=\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

37. Find the equation of the plane passing through the points (2, 2, –1) and (3, 4, 2) and parallel to the line whose direction ratios are 7, 0, 6.

OR

Consider the lines

$L_1 : \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$, $L_2 : \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

Find the distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2.

38. Solve the following LPP graphically.

Maximize, $Z = 150x + 250y$

Subject to the constraints

$x + y \leq 35$

$1000x + 2000y \leq 50000$

$x, y \geq 0$

OR

Find the number of point(s) at which the objective function $Z = 4x + 3y$ can be minimum subjected to the constraints $3x + 4y \leq 24$, $8x + 6y \leq 48$, $x \leq 5$, $y \leq 6$; $x, y \geq 0$.

1. Here, $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$

$\therefore$ Its magnitude $=|\vec{a}|$

$=\sqrt{3^2+(-2)^2+6^2}=\sqrt{9+4+36}=\sqrt{49}=7.$

OR

Given, $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$

Unit vectors perpendicular to $\vec{a}$ and $\vec{b}$ are $\pm\left(\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$.

So, there are two unit vectors perpendicular to the given vectors.

2. Let D_1, D_2 be the events that we find a defective fuse in the first and second test respectively.

$\therefore$ Required probability $=P(D_1\cap D_2)$

$=P(D_1)P(D_2|D_1)=\dfrac{2}{7}\cdot\dfrac{1}{6}=\dfrac{1}{21}$

3. For transitivity of a relation, if $(a, b)\in R$ and $(b, c)\in R \Rightarrow (a, c)\in R$

We have, $R=\{(1, 2), (2, 1)\}$

$\Rightarrow (1, 2)\in R$ and $(2, 1)\in R$ but $(1, 1)\notin R$

$\therefore$ R is not transitive.

OR

Here, $R=\{(a, b)\in A\times A : 2 \text{ divides } (a-b)\}$, which is an equivalence relation, where $A=\{0, 1, 2, 3, 4, 5\}$.

Clearly, $[0]=\{a\in A : aR0\}$

$=\{a\in A : 2 \text{ divides } (a-0)\}=\{a\in A : 2 \text{ divides } a\}$

$=\{0, 2, 4\}$

$\therefore$ Equivalence class $[0]$ is $\{0, 2, 4\}$.

4. The given equation of line is

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3} \quad \text{...(i)}$$

$$\Rightarrow \frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3} \Rightarrow \frac{x-4}{2}=\frac{y}{-6}=\frac{z-1}{3}$$

Now, as $\sqrt{2^2+(-6)^2+3^2}=7$

$\therefore$ D.c's. of (i) are $\dfrac{2}{7},\dfrac{-6}{7},\dfrac{3}{7}$.

5. Since, matrix A is skew symmetric matrix.

$\therefore$ $A'=-A$ ...(i)

Now, as $A=\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ $\therefore$ $A'=\begin{bmatrix}0 & 2 & b\\ a & 0 & 1\\ -3 & -1 & 0\end{bmatrix}$

From (i), $A+A'=O$

$$\Rightarrow \begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}+\begin{bmatrix}0 & 2 & b\\ a & 0 & 1\\ -3 & -1 & 0\end{bmatrix}=O$$

$$\Rightarrow \begin{bmatrix}0 & 2+a & b-3\\ a+2 & 0 & 0\\ b-3 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\therefore$ $a+2=0$ and $b-3=0$

$\Rightarrow$ $a=-2$ and $b=3$

OR

Here, $a_{ij}=\dfrac{|i-j|}{2}$

$\therefore$ For $i=2, j=3$ we have, $a_{23}=\dfrac{|2-3|}{2}=\dfrac{|-1|}{2}=\dfrac{1}{2}$

6. Expanding the given determinant, we get

$x(-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)+\cos\theta(-\sin\theta+x\cos\theta)=8$

$\Rightarrow -x^3-x+x=8 \Rightarrow x^3+8=0$

$\Rightarrow (x+2)(x^2-2x+4)=0$

$\Rightarrow x+2=0$ $\quad [\because x^2-2x+4>0\ \forall x]$

$\Rightarrow x=-2$

7. We have,

$$\int\frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx=\int(\sec^2x-\operatorname{cosec}^2x)dx$$

$=\tan x+\cot x+C$

OR

Let $I=\displaystyle\int_2^4\frac{x}{x^2+1}dx$

Put $x^2+1=t \Rightarrow 2xdx=dt \Rightarrow xdx=\dfrac{1}{2}dt$

Also $x=2\Rightarrow t=5$ and $x=4\Rightarrow t=17$

$$\therefore\ I=\frac{1}{2}\int_5^{17}\frac{dt}{t}=\frac{1}{2}[\log|t|]_5^{17}$$

$$=\frac{1}{2}[\log 17-\log 5]=\frac{1}{2}\log\left(\frac{17}{5}\right)$$

8. We have, $A=\{1, 2, 3\}$, $B=\{4, 5, 6, 7\}$ and $f=\{(1, 4), (2, 5), (3, 6)\}$

$\therefore$ $f(1)=4$, $f(2)=5$ and $f(3)=6$.

Since, distinct elements of A have distinct images in B, therefore, f is one-one function.

9. Consider, $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$
$= 2(\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3) - 3$ ($\because \cos 2x = 2\cos^2 x - 1$)
$= 2(1) - 3 = -1$

OR

Since direction ratios of line AB are 1, –2, 4, therefore the direction ratios of line parallel to AB will be proportional to 1, –2 and 4.

10. Given, $y - px = \sqrt{a^2p^2 + b^2}$
$\Rightarrow (y - px)^2 = a^2p^2 + b^2$
$\Rightarrow (x^2 - a^2)p^2 - 2xyp + (y^2 - b^2) = 0$
$\Rightarrow (x^2 - a^2)\left(\frac{dy}{dx}\right)^2 - 2xy\left(\frac{dy}{dx}\right) + (y^2 - b^2) = 0$

Hence, order is 1 and degree is 2.

11. We know that, the area enclosed by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq. units.

12. The given vectors are
$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$
$\therefore$ Required sum $= \vec{a} + \vec{b} + \vec{c}$
$= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k}) = -4\hat{j} - \hat{k}$.

13. We have, $R = \{(a, b) : a < b\}$, where $a, b \in \mathbb{R}$
(i) Symmetric : Let $(x, y) \in R$, i.e., $x R y \Rightarrow x < y$
Then, $y \nless x$, so $(x, y) \in R \Rightarrow (y, x) \notin R$
Thus, R is not symmetric.
(ii) Transitive : Let $(x, y) \in R$ and $(y, z) \in R$
$\Rightarrow x < y$ and $y < z \Rightarrow x < z$
$\Rightarrow (x, z) \in R$. Thus, R is transitive.

14. We have, $A^2 = A$
Now, L.H.S. $= (I - A)^3 + A = (I - A)(I - A)(I - A) + A$
$= (I \cdot I - I \cdot A - A \cdot I + A \cdot A)(I - A) + A$
$= (I - A - A + A)(I - A) + A$
$[\because I \cdot A = A \cdot I = A$ and $A^2 = A]$
$= (I - A)(I - A) + A$
$= (I \cdot I - I \cdot A - A \cdot I + A \cdot A) + A$
$= (I - A - A + A) + A = (I - A) + A = I =$ R.H.S.
Hence, $(I - A)^3 + A = I$

15. Given, $P(A | B) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$

$\Rightarrow P(A \cap B) = \frac{2}{5}P(B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$

Hence, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{1}{2} \times \frac{5}{13} + \frac{5}{13} - \frac{2}{13}$ $\left(\because 2P(A) = \frac{5}{13}, \therefore P(A) = \frac{1}{2} \times \frac{5}{13}\right)$
$= \frac{5 + 10 - 4}{26} = \frac{11}{26}$.

16. We have, $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 400$ and $|\vec{a}| = 4$
We know that, $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$
$\Rightarrow 400 = (4)^2 |\vec{b}|^2 \Rightarrow 16|\vec{b}|^2 = 400$
$\Rightarrow |\vec{b}|^2 = 25 \Rightarrow |\vec{b}| = 5$

17. Let A be the event that Neelam is selected and B be the event that Ved is selected. Then, we have,
$P(A) = \frac{1}{6}$
$\Rightarrow P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6} = P$ (Neelam is not selected)
$P(B) = \frac{1}{4}$
$\Rightarrow P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4} = P$ (Ved is not selected)

(i) (b) : P (both are selected) $= P(A \cap B) = P(A) \cdot P(B)$
$= \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

(ii) (c) : P (both are rejected) $= P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$
$= \frac{5}{6} \times \frac{3}{4} = \frac{5}{8}$

(iii) (d) : P (only one of them is selected)
$= P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$
$= \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4} = \frac{3}{24} + \frac{5}{24} = \frac{8}{24} = \frac{1}{3}$

(iv) (a) : P (at least one of them is selected)
$= 1 - P$ (Both are rejected)
$= 1 - \frac{5}{8} = \frac{3}{8}$

(v) (b) : Let E_1 be the event that Neelam is selected for post P and E_2 be the event that Neelam is selected for post Q.
$\therefore$ P (Neelam is selected for atleast one post)
$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$= \frac{1}{6} + \frac{1}{7} - \frac{1}{6} \times \frac{1}{7} = \frac{12}{42} = \frac{2}{7}$

18. (i) (b) : Since, side of square is of length 18 cm, therefore $x \in (0, 9)$.
(ii) (a) : Clearly, height of open box $= x$ cm
Length of open box $= 18 - 2x$
and width of open box $= 18 - 2x$
$\therefore$ Volume (V) of the open box $= x \times (18 - 2x) \times (18 - 2x)$
(iii) (d) : We have, $V = x(18 - 2x)^2$
$\therefore \frac{dV}{dx} = x \cdot 2(18 - 2x)(-2) + (18 - 2x)^2$
$= (18 - 2x)(-4x + 18 - 2x)$
$= (18 - 2x)(18 - 6x)$

Now, $\frac{dV}{dx}=0 \Rightarrow 18-2x=0$ or $18-6x=0$

$\Rightarrow x=9$ or 3

(iv) (c) : We have, $V=x(18-2x)^2$

and $\frac{dV}{dx}=(18-2x)(18-6x)$

$$\Rightarrow \frac{d^2V}{dx^2}=(18-2x)(-6)+(18-6x)(-2)$$
$$=(-2)[54-6x+18-6x]$$
$$=(-2)[72-12x]=24x-144$$

For $x=3$, $\frac{d^2V}{dx^2}<0$

and for $x=9$, $\frac{d^2V}{dx^2}>0$

So, volume will be maximum when $x=3$.

(v) (d) : We have, $V=x(18-2x)^2$, which will be maximum when $x=3$.

$\therefore$ Maximum volume $=3(18-6)^2$
$$=3\times 12\times 12=432 \text{ cm}^3$$

19. Let $u(x)=\sin^2 x$ and $v(x)=e^{\cos x}$.

Now, $\frac{du}{dx}=2\sin x\cos x$

and $\frac{dv}{dx}=e^{\cos x}(-\sin x)=-(\sin x)e^{\cos x}$

Thus, $\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{2\sin x\cos x}{-\sin x\, e^{\cos x}}=-\frac{2\cos x}{e^{\cos x}}$

20. Let $I=\int\frac{dx}{\sqrt{5-4x-2x^2}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{5}{2}-2x-x^2}}$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{7}{2}-1-2x-x^2}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2-(x+1)^2}}$$

$$=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x+1}{\sqrt{\frac{7}{2}}}\right)+C=\frac{1}{\sqrt{2}}\sin^{-1}\left[\sqrt{\frac{2}{7}}(x+1)\right]+C$$

OR

Let $I=\int\frac{\sin 2x}{\sin\left(x-\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)}dx$

$$=\int\frac{\sin\left\{\left(x-\frac{\pi}{3}\right)+\left(x+\frac{\pi}{3}\right)\right\}}{\sin\left(x-\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)}dx$$

$$=\int\frac{\left\{\sin\left(x-\frac{\pi}{3}\right)\cos\left(x+\frac{\pi}{3}\right)+\cos\left(x-\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)\right\}}{\sin\left(x-\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)}dx$$

$$=\int\left\{\cot\left(x+\frac{\pi}{3}\right)+\cot\left(x-\frac{\pi}{3}\right)\right\}dx$$

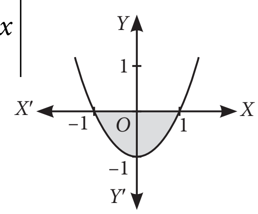
$$=\log\left|\sin\left(x+\frac{\pi}{3}\right)\right|+\log\left|\sin\left(x-\frac{\pi}{3}\right)\right|+C$$

21. The equation $y=x^2-1$ represents a upward parabola with vertex at $(0,-1)$.

It cuts x-axis where $y=0$

i.e., $x^2-1=0\Rightarrow x=\pm 1$

$\therefore$ Required area $=\left|\int_{-1}^{1}(x^2-1)\,dx\right|$



$$=\left|\left[\frac{x^3}{3}\right]_{-1}^{1}-[x]_{-1}^{1}\right|$$

$$=\left|\frac{1}{3}(1+1)-(1+1)\right|=\left|\frac{2}{3}-2\right|=\frac{4}{3} \text{ sq. units}$$

22. We know that the range of principal value branch of $\cos^{-1}$ and $\sin^{-1}$ are $[0,\pi]$ and $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ respectively.

Let $\cos^{-1}\left(\frac{1}{2}\right)=x \Rightarrow \frac{1}{2}=\cos x$

Then, $\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$, where $\frac{\pi}{3}\in[0,\pi]$

Let $\sin^{-1}\left(\frac{1}{2}\right)=y \Rightarrow \frac{1}{2}=\sin y$

Then, $\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$, where $\frac{\pi}{6}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore\ \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+2\cdot\frac{\pi}{6}=\frac{2\pi}{3}$.

23. Given, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 1 & 2 & 3\end{vmatrix}$

$=(9+2)\hat{i}-(6+1)\hat{j}+(4-3)\hat{k}=11\hat{i}-7\hat{j}+\hat{k}$.

$\therefore\ |\vec{a}\times\vec{b}|=\sqrt{(11)^2+(-7)^2+(1)^2}=\sqrt{171}$

OR

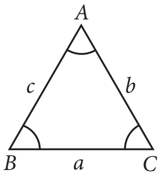
Area of the triangle ABC

$=\frac{1}{2}|\overrightarrow{BC}\times\overrightarrow{BA}|=\frac{1}{2}|ac\sin B\,\hat{n}|$

$= \frac{1}{2}ac\sin B = \frac{1}{2}a\frac{c}{\sin C}\sin B\sin C$

$= \frac{1}{2}a\frac{a}{\sin A}\sin B \sin C$ [By sine rule]

$= \frac{a^2 \sin B \sin C}{2\sin A}$



24. Here, $P(A)\cdot P(B) = \frac{1}{3}\times\frac{1}{4} = \frac{1}{12} = P(A\cap B)$

$\Rightarrow$ Events A and B are independent.

$\Rightarrow$ Events $\bar{A}$ and $\bar{B}$ are also independent.

Now, $P(\bar{A}\cap\bar{B}) = P(\bar{A})\,P(\bar{B})$

($\because$ $\bar{A}$ and $\bar{B}$ are independent events)

$= (1 - P(A))(1 - P(B))$

$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$

25. We have, $f(x) = 4x^3 - 18x^2 + 27x - 7$

$\Rightarrow f'(x) = 12x^2 - 36x + 27$

$= 12\left(x^2 - 3x + \frac{9}{4}\right) = 12\left(x-\frac{3}{2}\right)^2 \geq 0\ \forall x\in R$

Hence, $f(x)$ is always increasing on R.

26. We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$\Rightarrow (ax + 3)dx = (2y + f)dy$

$\Rightarrow a\frac{x^2}{2} + 3x = y^2 + fy + C$ (Integrating both sides)

$\Rightarrow -\frac{a}{2}x^2 + y^2 - 3x + fy + C = 0$

This will represent a circle, if $\frac{-a}{2} = 1 \Rightarrow a = -2$

[$\because$ coefficient of x^2 should be equal to coefficient of y^2]

27. Vector equation of the line passing through $(1, 2, -1)$ and parallel to the line

$5x - 25 = 14 - 7y = 35z$

i.e., $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$ or $\frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$

is $\vec{r} = (\hat{i}+2\hat{j}-\hat{k})+\lambda(7\hat{i}-5\hat{j}+\hat{k})$

28. Given that $|5\text{ adj}A| = 5$ and

A is non-singular matrix, *i.e.*, $|A| \neq 0$.

Clearly, $|5\text{ adj}A| = 5$

$\Rightarrow 5^3|\text{adj }A| = 5 \Rightarrow |\text{adj}A| = \frac{1}{5^2}$

$\Rightarrow |A|^{3-1} = \frac{1}{5^2}$ ($\because$ $|\text{adj }A| = |A|^{n-1}$, if $|A| \neq 0$)

$\Rightarrow |A| = \pm\frac{1}{5}$

OR

Given, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

Expanding along C_1, we get

$1(2a^2 + 4) - 2(-4a - 20) = 86$

$\Rightarrow 2a^2 + 4 + 8a + 40 = 86$

$\Rightarrow 2a^2 + 8a - 42 = 0$

$\Rightarrow a^2 + 4a - 21 = 0 \Rightarrow (a + 7)(a - 3) = 0 \Rightarrow a = 3, -7$

Now, sum of values of a is $3 + (-7) = 3 - 7 = -4$

29. Since, $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$ ...(i)

Here, $f(0) = \frac{2\times 0+1}{0-1} = -1$ ...(ii)

$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{2h+1}{h-1} = -1$ ...(iii)

and $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}$

$= \lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}\times\frac{\sqrt{1-kh}+\sqrt{1+kh}}{\sqrt{1-kh}+\sqrt{1+kh}}$

$= \lim_{h\to 0}\frac{(1-kh)-(1+kh)}{-h[\sqrt{1-kh}+\sqrt{1+kh}]}$

$= \lim_{h\to 0}\frac{2k}{\sqrt{1-kh}+\sqrt{1+kh}} = \frac{2k}{2} = k$ ...(iv)

Now, from (i), (ii), (iii) and (iv), we get $k = -1$

30. Let $I = \int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)dx$

$= \int_0^{\pi/2} 2\sin x\cos x\tan^{-1}(\sin x)dx$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

Also, $x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$\therefore I = \int_0^1 2t\tan^{-1}t\,dt = 2\int_0^1 t\tan^{-1}t\,dt$

Integrating by parts, we have

$I = 2\left[\frac{t^2}{2}\tan^{-1}t\right]_0^1 - 2\int_0^1\frac{t^2}{2(1+t^2)}dt$

$= 2\left[\frac{1}{2}\tan^{-1}1 - 0\right] - \int_0^1\frac{t^2}{(1+t^2)}dt$

$= 2\left[\frac{1}{2}\times\frac{\pi}{4}\right] - \int_0^1\frac{t^2+1-1}{(1+t^2)}dt = \frac{\pi}{4} - \int_0^1\left(1-\frac{1}{(1+t^2)}\right)dt$

$= \frac{\pi}{4} - [t]_0^1 + [\tan^{-1} t]_0^1$

$= \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1$

31. Let $y = \log\left(\frac{a + b\sin x}{a - b\sin x}\right)$

Then, $y = \log(a + b \sin x) - \log(a - b \sin x)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{a+b\sin x} \times \frac{d}{dx}(a + b\sin x) - \left\{\frac{1}{a - b\sin x} \times \frac{d}{dx}(a - b\sin x)\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{a+b\sin x}(0 + b\cos x) - \left\{\frac{1}{a-b\sin x}(0 - b\cos x)\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\cos x}{a+b\sin x} + \frac{b\cos x}{a - b\sin x}$$

$$\Rightarrow \frac{dy}{dx} = b\cos x\left\{\frac{1}{a+b\sin x} + \frac{1}{a-b\sin x}\right\}$$

$$\Rightarrow \frac{dy}{dx} = b\cos x\left\{\frac{a - b\sin x + a + b\sin x}{(a+b\sin x)(a-b\sin x)}\right\}$$

$$= \frac{2ab\cos x}{a^2 - b^2\sin^2 x}$$

OR

We have, $y = \sqrt{a + \sqrt{a + \sqrt{a + x^2}}}$, where a is a constant.

$$\Rightarrow y = \left[a + \sqrt{a + \sqrt{a + x^2}}\right]^{\frac{1}{2}}$$

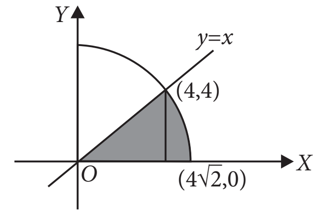
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left[a + \sqrt{a + \sqrt{a+x^2}}\right]^{-\frac{1}{2}} \frac{d}{dx}\left[a + \sqrt{a + \sqrt{a + x^2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left[a + \sqrt{a + \sqrt{a+x^2}}\right]^{-\frac{1}{2}} \left[\frac{1}{2}\left(a + \sqrt{a+x^2}\right)^{-\frac{1}{2}}\right] \times \frac{d}{dx}\left(a + \sqrt{a + x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left[a + \sqrt{a + \sqrt{a+x^2}}\right]^{-\frac{1}{2}} \left[\frac{1}{2}\left(a + \sqrt{a+x^2}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{2}(a + x^2)^{-\frac{1}{2}} \cdot 2x\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{4}x\left[\left(a + \sqrt{a + \sqrt{a + x^2}}\right) \cdot \left(a + \sqrt{a + x^2}\right) \cdot (a + x^2)\right]^{-\frac{1}{2}}$$

32. We have, $x^2 + y^2 = 32$ ...(i), a circle with centre (0, 0) and radius $4\sqrt{2}$, and $y = x$ ...(ii), a straight line. Solving (i) and (ii), we get point of intersection (4, 4) in the first quadrant.



$\therefore$ Required area $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= 8 + \left[16 \times \frac{\pi}{2}\right] - \left[2\sqrt{32 - 4^2} + 16\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$= 8 + 8\pi - 8 - 4\pi = 4\pi$ sq. units

33. We have, $y^2 = ax^3 + b$

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 3ax^2 \Rightarrow \frac{dy}{dx} = \frac{3ax^2}{2y}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = \left(\frac{3ax^2}{2y}\right)_{(2,3)} = 2a$$

So, equation of tangent at the point (2, 3) is

$y - 3 = 2a(x - 2)$

$\Rightarrow y = 2ax - 4a + 3$ ...(i)

But we are given that equation of tangent at (2, 3) is $y = 4x - 5$ ...(ii)

$\therefore$ On comparing (i) and (ii), we get

$2a = 4 \Rightarrow a = 2$

$\because$ Point (2, 3) lies on the curve $y^2 = ax^3 + b$,

$\therefore (3)^2 = (2)^3 a + b \Rightarrow 9 = 8a + b$

$\Rightarrow 9 = 8 \times 2 + b \Rightarrow b = -7$

34. We have, $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$ where $P = \frac{2x}{1+x^2}$ and $Q = \frac{4x^2}{1+x^2}$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1 + x^2)} = 1 + x^2$

So, the required solution is given by

$$y(1 + x^2) = \int \frac{4x^2}{1+x^2}(1 + x^2)\,dx + C$$

$$\Rightarrow y(1 + x^2) = 4\int x^2 dx + C$$

$$\Rightarrow \ y(1+x^2)=\frac{4x^3}{3}+C$$

Given that $y(0) = 0$

$\therefore \ 0(1 + 0) = 0 + C \Rightarrow C = 0$

Thus, $y=\dfrac{4x^3}{3(1+x^2)}$ is the required solution.

OR

We have, $\dfrac{dy}{dx}=\dfrac{x(2\log x+1)}{\sin y+y\cos y}$

$\Rightarrow$ $(\sin y + y\cos y)dy = x(2\log x + 1)dx$

On integrating both sides, we get

$$-\cos y + y\sin y - (-\cos y)$$
$$=2\left[\log x\times\frac{x^2}{2}-\int\frac{1}{x}\times\frac{x^2}{2}dx\right]+\frac{x^2}{2}+C$$

$$\Rightarrow y\sin y = x^2\log x-\frac{x^2}{2}+\frac{x^2}{2}+C$$

$\Rightarrow$ $y\sin y = x^2 \log x + C$

$\because$ When $x = 1$, $y = \dfrac{\pi}{2}$

$$\therefore \ \frac{\pi}{2}\sin\frac{\pi}{2}=1\cdot\log(1)+C \ \Rightarrow \frac{\pi}{2}=C$$

$\therefore$ $y\sin y = x^2 \log x + \pi/2$ is the required solution.

35. Given, $f(x)=\dfrac{x}{1+|x|}, x\in(-\infty,0)$

$$=\frac{x}{1-x} \qquad (\because x \in (-\infty, 0), |x| = -x)$$

For one-one : Let $f(x_1) = f(x_2)$, $x_1x_2 \in (-\infty , 0)$

$$\Rightarrow \frac{x_1}{1-x_1}=\frac{x_2}{1-x_2}$$

$\Rightarrow$ $x_1(1 - x_2) = x_2(1 - x_1)$

$\Rightarrow$ $x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$\therefore$ f is one-one

For onto : Let $f(x) = y$

$$\Rightarrow y=\frac{x}{1-x} \Rightarrow y(1-x)=x \Rightarrow y-xy=x$$

$$\Rightarrow x+xy=y \Rightarrow x(1+y)=y \Rightarrow x=\frac{y}{1+y}$$

Here, $y \in (-1, 0)$

So, x is defined for all values of y.

Also $x \in (-\infty, 0)$ for all $y \in (-1, 0)$.

$\therefore$ f is onto.

36. We have,

$$|A|=\begin{vmatrix}8&4&2\\2&9&4\\1&2&8\end{vmatrix}=8(72-8)-4(16-4)+2(4-9)$$

$= 512 - 48 - 10 = 454 \neq 0$

Thus, A is a non-singular matrix and therefore it is invertible.

Let C_{ij} be cofactor of a_{ij} in A. Then,

$$C_{11}=\begin{vmatrix}9&4\\2&8\end{vmatrix}=64,\ C_{12}=-\begin{vmatrix}2&4\\1&8\end{vmatrix}=-12,\ C_{13}=\begin{vmatrix}2&9\\1&2\end{vmatrix}=-5$$

$$C_{21}=-\begin{vmatrix}4&2\\2&8\end{vmatrix}=-28,\ C_{22}=\begin{vmatrix}8&2\\1&8\end{vmatrix}=62,$$

$$C_{23}=-\begin{vmatrix}8&4\\1&2\end{vmatrix}=-12,$$

$$C_{31}=\begin{vmatrix}4&2\\9&4\end{vmatrix}=-2,\ C_{32}=-\begin{vmatrix}8&2\\2&4\end{vmatrix}=-28,\ C_{33}=\begin{vmatrix}8&4\\2&9\end{vmatrix}=64$$

$$\therefore \ \text{adj}\,A=\begin{bmatrix}64&-12&-5\\-28&62&-12\\-2&-28&64\end{bmatrix}'=\begin{bmatrix}64&-28&-2\\-12&62&-28\\-5&-12&64\end{bmatrix}$$

$$\text{Hence, } A^{-1}=\frac{1}{|A|}\text{adj}\,A=\frac{1}{454}\begin{bmatrix}64&-28&-2\\-12&62&-28\\-5&-12&64\end{bmatrix}$$

OR

$$\text{Here, } A=\begin{bmatrix}2&3\\1&-4\end{bmatrix} \text{ and } B=\begin{bmatrix}1&-2\\-1&3\end{bmatrix}$$

$\Rightarrow$ $|A| = -11$ and $|B| = 1$

$$\therefore \ A^{-1}=\frac{1}{|A|}\text{adj}\,A=-\frac{1}{11}\begin{bmatrix}-4&-3\\-1&2\end{bmatrix} \text{ and } B^{-1}=\begin{bmatrix}3&2\\1&1\end{bmatrix}$$

$\Rightarrow$ R.H.S. $= B^{-1} A^{-1}$

$$=\begin{bmatrix}3&2\\1&1\end{bmatrix}\cdot\left(-\frac{1}{11}\right)\begin{bmatrix}-4&-3\\-1&2\end{bmatrix}=\left(-\frac{1}{11}\right)\begin{bmatrix}-14&-5\\-5&-1\end{bmatrix} \quad \text{...(i)}$$

$$\text{Now, } A\cdot B=\begin{bmatrix}2&3\\1&-4\end{bmatrix}\cdot\begin{bmatrix}1&-2\\-1&3\end{bmatrix}=\begin{bmatrix}-1&5\\5&-14\end{bmatrix}$$

$\Rightarrow$ $|AB| = 14 - 25 = -11$

$$\therefore \ \text{L.H.S.} = (AB)^{-1}=\left(-\frac{1}{11}\right)\begin{bmatrix}-14&-5\\-5&-1\end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), $(AB)^{-1} = B^{-1} A^{-1}$.

37. The equation of a plane passing through $(2, 2, -1)$ is $a(x - 2) + b(y - 2) + c(z + 1) = 0$ ...(i)

This plane also passes through $(3, 4, 2)$.

$\therefore$ $a(3 - 2) + b(4 - 2) + c(2 + 1) = 0$

$\Rightarrow$ $a + 2b + 3c = 0$ ...(ii)

Now, plane (i) is parallel to the line whose direction ratios are 7, 0, 6

Therefore, $7a + 0(b) + 6c = 0$ ...(iii)

Solving (ii) and (iii) by cross-multiplication method, we get

$$\frac{a}{(2)(6)-(0)(3)}=\frac{b}{(7)(3)-(6)(1)}=\frac{c}{(0)(1)-(2)(7)}$$

$\Rightarrow \frac{a}{12}=\frac{b}{15}=\frac{c}{-14}=\lambda$ (say)

$\Rightarrow a = 12\lambda, b = 15\lambda, c = -14\lambda$

Substituting the values of a, b, c in (i), we get

$12\lambda(x-2)+15\lambda(y-2)-14\lambda(z+1)=0$

$\Rightarrow 12x-24+15y-30-14z-14=0$ $[\because \lambda \neq 0]$

$\Rightarrow 12x+15y-14z=68$

This is the required equation of plane.

OR

Any plane through $(-1, -2, -1)$ is

$a(x+1)+b(y+2)+c(z+1)=0$ ...(i)

D.R.'s of any normal to (i) are $< a, b, c >$.

As this normal is perpendicular to both L_1 and L_2, therefore,

$3a+1b+2c=0$ ...(ii)

$1a+2b+3c=0$ ...(iii)

Eliminating a, b, c between (i), (ii) and (iii), we obtain

$$\begin{vmatrix} x+1 & y+2 & z+1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}=0$$

$\Rightarrow (x+1)(3-4)-(y+2)(9-2)+(z+1)(6-1)=0$

$\Rightarrow -(x+1)-7(y+2)+5(z+1)=0$

or $x+7y-5z+10=0$ ...(iv)

This is the required equation of plane.

$\therefore$ Distance of (1, 1, 1) from the plane (iv)

$$=\frac{|1+7-5+10|}{\sqrt{1^2+7^2+(-5)^2}}=\frac{13}{\sqrt{75}} \text{ units.}$$

38. The given problem is

Maximize , $Z = 150x + 250y$

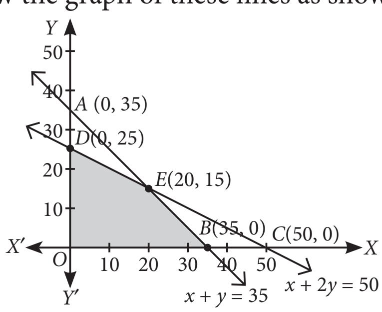
Subject to the constraints

$x + y \le 35, x + 2y \le 50$ and $x, y \ge 0$

To solve graphically, we convert the inequations into equations to obtain the following lines :

$x + y = 35, x + 2y = 50, x = 0, y = 0$

Let us draw the graph of these lines as shown below.



The feasible region is the shaded region. We observe that the region is bounded.

The corner points of the feasible region $OBED$ are $O(0, 0)$, $B(35, 0)$, $E(20, 15)$ and $D(0, 25)$.

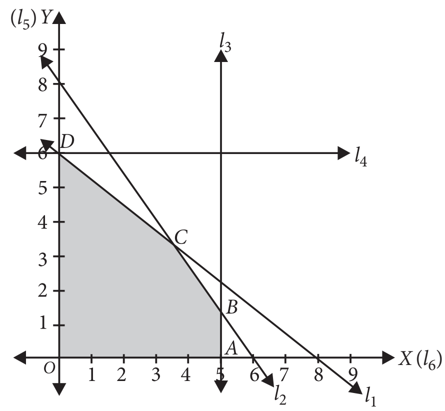
The value of the objective function at corner points of the feasible region are :

Corner points	Value of $Z = 150x + 250y$
$O(0, 0)$	0
$B(35, 0)$	5250
$E(20, 15)$	6750 (Maximum)
$D(0, 25)$	6250

Clearly, Z is maximum at $x = 20, y = 15$.

OR

Let $l_1 : 3x + 4y = 24$, $l_2 : 8x + 6y = 48$, $l_3 : x = 5$, $l_4 : y = 6$, $l_5 : x = 0$, $l_6 : y = 0$



For B : Solving l_2 and l_3, we get $B(5, 4/3)$

For C : Solving l_1 and l_2, we get $C\left(\frac{24}{7}, \frac{24}{7}\right)$

Shaded portion $OABCD$ is the feasible region, where $O(0, 0)$, $A(5, 0)$, $B(5, 4/3)$, $C\left(\frac{24}{7}, \frac{24}{7}\right)$ and $D(0, 6)$

Now minimize $Z = 4x + 3y$

Z at $O(0, 0) = 0$

Z at $A(5, 0) = 4 \times 5 + 3 \times 0 = 20$

Z at $B\left(5, \frac{4}{3}\right) = 4\times5+3\times\frac{4}{3} = 24$

Z at $C\left(\frac{24}{7}, \frac{24}{7}\right) = 4\times\frac{24}{7}+3\times\frac{24}{7} = 24$

Z at $D(0, 6) = 4 \times 0 + 3 \times 6 = 18$

Thus, Z is minimum at 1 point $O(0, 0)$.

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